

A New Similarity Measure for Vague Sets

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Abstract--Similarity measure is one of important, effective and widely-used methods in data processing and analysis. As vague set theory has become a promising representation of fuzzy concepts, in this paper we present a similarity measure approach for better understanding the relationship between two vague sets in applications. Compared to existing similarity measures, our approach is far more reasonable, practical yet useful in measuring the similarity between vague sets.

Index Terms-- Fuzzy sets, Vague sets, Similarity measure

I. INTRODUCTION

In the classical set theory introduced by Cantor, a German mathematician, values of elements in a set are only one of 0 and 1. That is, for any element, there are only two possibilities: in or not in the set. Therefore, the theory cannot handle the data with ambiguity and uncertainty.

Zadeh proposed fuzzy theory in 1965 [1]. The most important feature of a fuzzy set is that fuzzy set A is a class of objects that satisfy a certain (or several) property. Each object x has a membership degree of A , denoted as $\mu_A(x)$. This membership function has the following characteristics: The single degree contains the evidences for both supporting and opposing x . It cannot only represent one of the two evidences, but it cannot represent both at the same time too.

In order to deal with this problem, Gau and Buehrer proposed the concept of vague set in 1993 [2], by replacing the value of an element in a set with a sub-interval of $[0, 1]$. Namely, a true-membership function $t_v(x)$ and a false-membership function $f_v(x)$ are used to describe the boundaries of membership degree. These two boundaries form a sub-interval $[t_v(x), 1 - f_v(x)]$ of $[0, 1]$. The vague set theory improves description of the objective real world, becoming a promising tool to deal with inexact, uncertain or vague knowledge. Many researchers have applies this theory to many situations, such as fuzzy control, decision-making, knowledge discovery and fault diagnosis. And the tool has presented more challenging than that with fuzzy sets theory in applications.

In intelligent activities, it is often needed to compare and couple between two fuzzy concepts. That is, we need to check whether two knowledge patterns are identical or approximately same, to find out functional dependence relations between concepts in a data mining system. Many measure methods have been proposed to measure the similarity between two vague sets (values). Each of them is given from different side, having its own counterexamples. Such as Shyi-Ming Chen proposed a similarity measure M_C in [3], whereas from the M_C model we can gain the similarity of vague values $[0.5, 0.5]$ and $[0, 1]$ is 1, obviously

their similarity should not be 1. In reference [5] Dug Hun Hong put forward another similarity measure M_H , according to the formulae of M_H , we can obtain the similarity of vague values $[0.3, 0.7]$ and $[0.4, 0.6]$ is $M_H([0.3,0.7],[0.4,0.6]) = 0.9$, in the same model, we can get $M_H([0.3,0.6],[0.4,0.7]) = 0.9$. In a voting model, the vague value $[0.3, 0.7]$ can be interpreted as: "the vote for a resolution is 3 in favor, 3 against and 4 abstentions"; $[0.4, 0.6]$ can be interpreted as: "the vote for a resolution is 4 in favor, 4 against and 2 abstention". $[0.3, 0.6]$ and $[0.4, 0.7]$ can have similar interpretation. Intuitively, $[0.3, 0.7]$ and $[0.4, 0.6]$ may be more similar than $[0.3, 0.6]$ and $[0.4, 0.7]$. Therefore sometimes the results of M_H model are not accordant with our intuition. After analyzing most existing vague sets and vague values similarity measures, we find out that almost each measure has its defect. In section 2.2, we will illustrate them with more examples.

Then we have to make a choice according to the applications. A more reasonable approach is proposed to measure similarity in this paper, after analyzing existing methods.

The remaining of this paper is organized as follows. In Section 2, several methods of similarity measure for vague set are discussed. An improved similarity measure method and its properties are given in Section 3. Section 4 concludes this paper.

II. PRELIMINARIES

A. Vague set

In this section, we review some basic definitions of vague values and vague sets from [2], [3], [4].

Definition 1 Vague Sets [2]: Let X be a space of points (objects), with a generic element of X denoted by x . A vague set V in X is characterized by a truth-membership function t_v and a false-membership function f_v . t_v is a lower bound on the grade of membership of x derived from the evidence for x , and f_v is a lower bound on the negation of x derived from the evidence against x , t_v and f_v both associate a real number in the interval $[0,1]$ with each point in X , where $t_v + f_v \leq 1$. That is

$$t_v : X \rightarrow [0,1] ; f_v : X \rightarrow [0,1]$$

This approach bounds the grade of membership of x to a sub-interval $[t_v(x), 1 - f_v(x)]$ of $[0,1]$

When X is continuous, a vague set V can be written as

$$V = \int_X [t_v(x), 1 - f_v(x)] / x, x \in X.$$

When X is discrete, a vague set V can be written as

$$V = \sum_{i=1}^n [t_v(x_i), 1 - f_v(x_i)] / x_i, x_i \in X.$$

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Definition 2: Let x and y be two vague values, where $x = [t_x, 1 - f_x]$ and $y = [t_y, 1 - f_y]$. If $t_x = t_y$ and $f_x = f_y$, then the vague values x and y are called equal (i.e., $[t_x, 1 - f_x] = [t_y, 1 - f_y]$).

Definition 3: Let A and B be vague sets of the universe of discourse U , $U = \{u_1, u_2, \dots, u_n\}$, where

$$A = [t_A(u_1), 1 - f_A(u_1)]/u_1 + [t_A(u_2), 1 - f_A(u_2)]/u_2 + \dots + [t_A(u_n), 1 - f_A(u_n)]/u_n$$

$$B = [t_B(u_1), 1 - f_B(u_1)]/u_1 + [t_B(u_2), 1 - f_B(u_2)]/u_2 + \dots + [t_B(u_n), 1 - f_B(u_n)]/u_n$$

If $\forall i, [t_A(u_i), 1 - f_A(u_i)] = [t_B(u_i), 1 - f_B(u_i)]$, then the vague sets A and B are called equal, where $1 \leq i \leq n$.

B. Research into similarity measure

Currently, there have been many similarity measurements for vague set (value). Suppose that $X = [t_x, 1 - f_x]$ and $Y = [t_y, 1 - f_y]$ are two vague values over the discourse universe U . Let $S(x) = t_x - f_x$, $S(y) = t_y - f_y$, the M_C , M_H , M_L and M_O models are defined respectively in [3], [5], [6] and [7] as follows,

$$M_C(x, y) = 1 - \frac{|S(x) - S(y)|}{2}$$

$$= 1 - \frac{|(t_x - t_y) - (f_x - f_y)|}{2} \tag{1}$$

$$M_H(x, y) = 1 - \frac{|t_x - t_y| + |f_x - f_y|}{2} \tag{2}$$

$$M_L(x, y) = 1 - \frac{|S(x) - S(y)|}{4} - \frac{|t_x - t_y| + |f_x - f_y|}{4}$$

$$= 1 - \frac{|(t_x - t_y) - (f_x - f_y)| + |t_x - t_y| + |f_x - f_y|}{4} \tag{3}$$

$$M_O(x, y) = 1 - \sqrt{\frac{(t_x - t_y)^2 + (f_x - f_y)^2}{2}} \tag{4}$$

Comparisons among the M_C , M_H , M_L , M_O models can also be found in [7].

From the definition of the M_C model, we know that

$$t_x - f_x = t_y - f_y \Rightarrow M_C \equiv 1,$$

i.e. the M_C model is too rough when $t_x - f_x = t_y - f_y$.

The M_H model pays equal attention both to the difference of two true-membership degrees and to the difference of two false-membership degrees, between two vague values. Pairs of vague values, which have both the same difference of true-membership degrees and the same difference of false-membership degrees, have the same similarity. But it does not distinguish the positive difference and negative difference between true- and false-membership degrees.

The M_L model inherits the advantages of the M_C and M_H models, paying equal attentions to the support of vague value, true-membership degree, and false-membership degree, respectively. But it uses absolute values, and hence increases the possibility of similarity coincidence. For example, it cannot distinguish between pair of $([0.4, 0.8], [0.5, 0.7])$ and pair of $([0.4, 0.8], [0.5,$

$0.8])$. According to our intuition, pair of $([0.4, 0.8], [0.5, 0.8])$ is more similar than pair of $([0.4, 0.8], [0.5, 0.7])$, but in the model of M_L , the two pairs of vague values have the same similarity.

The M_O model also reflects the equal concerns between the difference of true-membership degrees and the difference of false-membership degrees. But similar to the M_H model, the M_O model does not consider whether the differences are positive or negative.

The above methods of similarity measure can be used to solve the problem of how to determine the similarity between two vague values in a certain extent. But each of them focuses on different aspects. There are three factors which affect the similarity of vague values: true-membership function t_x , false-membership function f_x , and $1 - t_x - f_x$. The reason there are many counterexamples under the measures of the M_H , M_L and M_O models is that the weights of $|t_x - t_y|$, $|f_x - f_y|$ and $|(t_y + f_y) - (t_x + f_x)|$ in the above methods are constants. Its explicit characteristic is that it is not considered whether the difference is positive or negative. Based on this idea, a new weighted and variable similarity measure is proposed. It can considerably reduce the possibility of similarity coincidence.

III. MEASURING THE SIMILARITY BETWEEN VAGUE SETS

This section constructs a new approach for measuring the similarity between vague sets, analyzes the properties and illustrates the use by examples.

A. A New Similarity Measure

We first give an example. Assume that there are four candidates A, B, C, D , and ten voters. One voter supports A , one opposes A ; two support B , one opposes B ; seven support C , one opposes C ; eight support D , and one opposes D . The voting results of A, B, C , and D can be viewed as four vague values, $A[0.1, 0.9]$, $B[0.2, 0.9]$, $C[0.7, 0.9]$, and $D[0.8, 0.9]$. Now we compare the similarities between A and B , and between C and D . By formulae (1), (2), (3) and (4), we obtain the similarities as shown in Table 1.

TABLE 1.
AN EXAMPLE OF SIMILARITY CALCULATION

	x	y	M_C	M_H	M_L	M_O	M'
A, B	$[0.1, 0.9]$	$[0.2, 0.9]$	0.95	0.95	0.95	0.929	0.968
C, D	$[0.7, 0.9]$	$[0.8, 0.9]$	0.95	0.95	0.95	0.929	0.953

From Table 1, we can see that the similarities between A and B (and between C and D) are all the same by using the M_C , M_H , M_L and M_O models. If only a candidate can be selected and renunciation is considered, D is most possible to be selected. The possibility of selecting C is smaller than that of selecting D . Selecting A or B has rather low possibility. Intuitively, it should be easier to say that A and B are similar than C and D , because A and B are all impossible options, D might be selected, and C might not be selected. Among A, B, C , and D , we would be most concerned with the similarity between C and D . We need to enlarge the difference of similarities where we are concerned.

For another example, assume that there are other four candidates E, F, G, H , and ten voters. One voter supports E , nine op-

pose E ; one supports F , eight oppose F ; one supports G , two oppose G ; one supports H , and one opposes H . The voting results of E, F, G , and H can also be viewed as four vague values, $E[0.1, 0.1]$, $F[0.1, 0.2]$, $G[0.1, 0.8]$, and $H[0.1, 0.9]$. Now we compare the similarities between E and F , and between G and H . By formulae (1), (2), (3) and (4), we obtain the similarities as shown in Table 2.

TABLE 2.
AN EXAMPLE OF SIMILARITY CALCULATION

	x	y	M_C	M_H	M_L	M_O	M'
E, F	[0.1, 0.1]	[0.1, 0.2]	0.95	0.95	0.95	0.929	0.948
G, H	[0.1, 0.8]	[0.1, 0.9]	0.95	0.95	0.95	0.929	0.931

We definitely know only the minority of voters support the candidates E, F, G or H , and the majority of voters oppose E or F , whereas we have little information about G or H because there are so many abstainers. Intuitively compared with E and F , G and H should have less similarity. But from the Table 2, we can see the similarities between E and F (and between G and H) are all the same by using the M_C, M_H, M_L and M_O models.

Then we need a new similarity measure which can magnify what we are concerned. That is, if $t_x - t_y$ is equal to $f_x - f_y$, the similarities can still be different. For example, the larger the support ($t_x + t_y$) is, the smaller the similarity should be. Analogically, the smaller the opposition ($f_x + f_y$) is, the smaller the similarity should be.

Based on the above discussion, we propose a weight-varied similarity measure M' , i.e.

$$M' = 1 - \frac{(t_x + t_y) |t_x - t_y|}{(t_x + t_y) + (2 - f_x - f_y) + 2} - \frac{(2 - f_x - f_y) |f_x - f_y|}{(t_x + t_y) + (2 - f_x - f_y) + 2} - \frac{|(t_x - t_y) - (f_x - f_y)|}{(t_x + t_y) + (2 - f_x - f_y) + 2} \quad (5)$$

Coefficient $(t_x + t_y)$ of $|t_x - t_y|$ implies that when $|t_x - t_y|$ is the same, similarity should be smaller if $(t_x + t_y)$ is larger. Coefficient $(2 - f_x - f_y)$ of $|f_x - f_y|$ means that when $|f_x - f_y|$ is the same, similarity should be smaller if $(f_x + f_y)$ is smaller.

Let $t_x + t_y = p$, $t_x - t_y = q$, $f_x + f_y = m$, $f_x - f_y = n$. Then, formula (5) can be reduced as

$$M' = 1 - \frac{|pq| + |(2 - m)n| + |q - n|}{p + (2 - m) + 2} \quad (6)$$

Where the M' model pays attention to both the difference of true-membership degrees and the difference of false-membership degrees between vague values. It also implies the attention to the support of vague value. Because of the introduction of $|(t_x - t_y) - (f_x - f_y)|$, The M' model can distinguish positive difference and negative difference. The strategy of varied-weight leads to the reduced possibility of similarity coincidence, and the weights meet the requirement that we are concerned with those similarities where supports are high and oppositions are low. For the sake of comparison, we enlarge the difference of similarities

when the support is large and the opposition is small.

From the above definition, we obtain the following properties.

Property 1: $M'(x, y) \in [0, 1]$.

Proof: Since $t_x \in [0, 1]$, $t_y \in [0, 1]$, $f_x \in [0, 1]$, $f_y \in [0, 1]$, we have

$$|t_x - t_y| \in [0, 1], |f_x - f_y| \in [0, 1], |(t_x - t_y) - (f_x - f_y)| \in [0, 2]$$

$$M' \leq 1 - \frac{(t_x + t_y) \cdot 0 + (2 - f_x - f_y) \cdot 0 + 0}{(t_x + t_y) + (2 - f_x - f_y) + 2} = 1$$

$$M' \geq 1 - \frac{(t_x + t_y) + (2 - f_x - f_y) + 2}{(t_x + t_y) + (2 - f_x - f_y) + 2} = 0 \quad \blacksquare$$

Property 2: $M'(x, y) = M'(y, x)$.

It is obtained directly from the definition of the M' model.

Property 3: $M'(x, y) = 0 \Leftrightarrow x = [0, 0]$ and $y = [1, 1]$; or $x = [1, 1]$ and $y = [0, 0]$.

Proof: For $x = [0, 0]$ and $y = [1, 1]$ (or for $x = [1, 1]$ and $y = [0, 0]$), by the definition, we obviously have $M'(x, y) = 0$; and

If $M'(x, y) = 0$, we have $t_x - t_y = 1$ and $f_x - f_y = -1$;

$$\text{or } t_x - t_y = -1, f_x - f_y = 1$$

Hence, $x = [0, 0]$ and $y = [1, 1]$;

$$\text{or } x = [1, 1], y = [0, 0] \quad \blacksquare$$

Property 4: $M'(x, y) = 0 \Leftrightarrow x = y$.

Proof: If $x = y$, from the definition, it is clear that

$$M'(x, y) = 1.$$

If $M'(x, y) = 1 \Rightarrow t_x - t_y = 0, f_x - f_y = 0$, that is, $x = y$. \blacksquare

Example 1: In table 3, seven groups of vague values (x, y) are given. Intuitively, the similarity of the first pair vague values should be larger than the second pair, namely $M(x_1, y_1) > M(x_2, y_2)$. And experientially $M(x_4, y_4) < M(x_5, y_5)$; $M(x_6, y_6) < M(x_7, y_7)$. Consider the 7 groups of data pairs (x, y) in the second and third rows of Table 2. We compare our measure method with others. The results are shown in 4th — 8th rows of Table 3.

TABLE 3.
COMPARISONS OF VARIOUS SIMILARITY MEASURES

	x	y	M_C	M_H	M_L	M_O	M'
1	[0.3, 0.7]	[0.4, 0.6]	1	0.9	0.9	0.9	0.95
2	[0.3, 0.6]	[0.4, 0.7]	0.9	0.9	0.9	0.9	0.9
3	[0.3, 0.8]	[0.4, 0.7]	1	0.9	0.9	0.9	0.94
4	[1, 1]	[0, 1]	0.5	0.5	0.5	0.3	0.6
5	[0.5, 0.5]	[0, 1]	1	0.5	0.7	0.5	0.75
6	[0.4, 0.8]	[0.5, 0.7]	1	0.9	0.9	0.9	0.94
7	[0.4, 0.8]	[0.5, 0.8]	0.9	0.9	0.9	0.92	0.95

From the Table 3, we can see sometimes the similarities gained by formulae of M_C, M_H, M_L and M_O are counterintuitive.

For example, $M_C(x_1, y_1) = M_C([0.3, 0.7], [0.4, 0.6]) = 1$, apparently we know the similarity of $[0.3, 0.7]$ and $[0.4, 0.6]$ is absolutely not 1.

Another example, compare the similarity of the first group data pair ([0.3, 0.7], [0.4, 0.6]) with the second group data pair ([0.3, 0.6], [0.4, 0.7]) using the several different similarity measures. We get

$$M_H(x_1, y_1) = M_H(x_2, y_2), M_O(x_1, y_1) = M_O(x_2, y_2),$$

whereas intuitively the first data pair should be more similar than the second data pair, namely $M(x_1, y_1) > M(x_2, y_2)$, but only M_L and M' can be accordant with our intuition — $M(x_1, y_1) > M(x_2, y_2)$.

Then compare the similarity of the sixth group data pair ([0.4, 0.8], [0.5, 0.7]) with the seventh group data pair ([0.4, 0.8], [0.5, 0.8]). Intuitively, the similarity of the sixth group and the seventh group should satisfy $M(x_6, y_6) < M(x_7, y_7)$, but from the above result we can see only M_H, M_O , and M' can satisfy the limitation.

To sum up, none but M' can distinguish those groups vague values, to some extent, according with our intuition.

In table 3, we give more comparison of different similarity measures.

In the new similarity measure M' , the three factors are considered equally which affect the similarity of vague values: true-membership function t_x , false-membership function f_x , and $1 - t_x - f_x$. The weights of $|t_x - t_y|$, $|f_x - f_y|$ and $|(t_y + f_y) - (t_x + f_x)|$ in new similarity measure M' are variable, and the variable weights reduce the possibility of similarity coincidence. Simultaneously, the new similarity measure M' enlarges the difference of similarities where we are concerned then it is easier for us to do some decision.

B. Similarity measure between Vague Sets

Assume that A and B are two vague sets over the discourse universe $U = \{u_1, u_2, \dots, u_n\}$. $V_A(u_i) = [t_A(u_i), 1 - f_A(u_i)]$ is the membership value of u_i in vague set A , and $V_B(u_i) = [t_B(u_i), 1 - f_B(u_i)]$ is the membership value of u_i in vague set B . Let

$$A = \sum_{i=1}^n [t_A(u_i), 1 - f_A(u_i)] / u_i$$

$$B = \sum_{i=1}^n [t_B(u_i), 1 - f_B(u_i)] / u_i$$

Then, the similarity between vague sets A and B can be obtained by the following function T' .

$$\begin{aligned} T'(A, B) &= \frac{1}{n} \sum_{i=1}^n M'(V_A(u_i), V_B(u_i)) \\ &= \frac{1}{n} \sum_{i=1}^n \left(1 - \frac{|p(u_i)q(u_i)| + |(2 - m(u_i))n(u_i)| + |q(u_i) - n(u_i)|}{p(u_i) + (2 - m(u_i)) + 2} \right) \end{aligned} \quad (7)$$

where,

$$t_x(u_i) + t_y(u_i) = p(u_i)$$

$$t_x(u_i) - t_y(u_i) = q(u_i)$$

$$f_x(u_i) + f_y(u_i) = m(u_i)$$

$$f_x(u_i) - f_y(u_i) = n(u_i)$$

From the above definition, we have the following properties.

Property 5: $T'(A, B) \in [0, 1]$.

Property 6: $T'(A, B) = T'(B, A)$.

Property 7:

$$T'(A, B) = 0 \Leftrightarrow A = \sum_{i=1}^n [0, 0] / u_i, B = \sum_{i=1}^n [1, 1] / u_i$$

$$\text{or } A = \sum_{i=1}^n [1, 1] / u_i, B = \sum_{i=1}^n [0, 0] / u_i$$

Property 8: $T'(A, B) = 1 \Leftrightarrow A = B$.

Example 2: Let A and B be two vague sets over the discourse universe $U = \{u_1, u_2, u_3, u_4\}$, where

$$A = [0.3, 0.7] / u_1 + [0.5, 0.5] / u_2 + [0.4, 0.8] / u_3 + [1.0, 1.0] / u_4$$

$$B = [0.4, 0.6] / u_1 + [0.0, 1.0] / u_2 + [0.5, 0.7] / u_3 + [0.0, 1.0] / u_4$$

From formula (7), we have the following similarity between A and B .

$$\begin{aligned} T'(A, B) &= \frac{1}{4} \sum_{i=1}^4 M'(V_A(u_i), V_B(u_i)) \\ &= [(1 - 0.05) + (1 - 0.25) + (1 - 0.055) + (1 - 0.4)] / 4 \\ &= 0.811 \end{aligned}$$

C. Weighted Similarity Measure between Vague Sets

Suppose that A and B are two vague sets over the discourse universe $U = \{u_1, u_2, \dots, u_n\}$, w_i is the weight of u_i , $w_i \in [0, 1]$, $1 \leq i \leq n$. Then, the weighted similarity between A and B can be obtained by calculating the following $W(A, B)$.

$$\begin{aligned} W'(A, B) &= \sum_{i=1}^n w_i M'(V_A(u_i), V_B(u_i)) / \sum_{i=1}^n w_i \\ &= \sum_{i=1}^n w_i \left(1 - \frac{|p(u_i)q(u_i)| + |(2 - m(u_i))n(u_i)| + |q(u_i) - n(u_i)|}{p(u_i) + (2 - m(u_i)) + 2} \right) / \sum_{i=1}^n w_i \end{aligned} \quad (8)$$

Example 3: Let A and B be the same as that in Example 2, the weights of elements u_1, u_2, u_3 , and u_4 in discourse universe U are 0.4, 0.2, 0.8, and 0.6, respectively. From (8), we have the weighted similarity between A and B

$$\begin{aligned} W'(A, B) &= \frac{0.4(1 - 0.05) + 0.2(1 - 0.25) + 0.8(1 - 0.055) + 0.6(1 - 0.4)}{0.4 + 0.2 + 0.8 + 0.6} \\ &= (0.38 + 0.15 + 0.756 + 0.36) / 2.0 \\ &= 0.823 \end{aligned}$$

IV. CONCLUSIONS

After analyzing the limitations in current similarity measures for vague sets, we have proposed a new method for measuring the similarity between vague sets in this paper. The basic idea is to deeply understand the support, the difference of true-membership and the difference of false-membership, to significantly distinguish the directions of difference (positive and negative), and properly use varied-weights in the differences of true- and false-membership, for two vague sets. The examples have illustrated that our approach is effective and practical, and presents much better discernibility than existing ones at measuring

the similarity between vague sets.

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